Impulse control methods in portfolio analysis of Levy noise asset prices models with obligatory diversification

Łukasz Stettner, Institute of Mathematics
Polish Academy of Sci., Warsaw, Poland, E-mail: stettner@impan.gov.pl

We assume the vector of asset prices are of the form

$$S_i(t) = S_i(0)e^{X_i(t)},$$

where $X_i(0) = 0$ and $X(t) = (X_1(t), \dots, X_d(t))^T$ with T standing for transpose, is a solution to the following Lévy stochastic differential equation:

$$dX(t) = a(z(t))\,dt + \sigma(z(t))dB(t) \ + \int_{R^d} \gamma(z(t), y)\,\tilde{N}(dt, dy)$$

with $(z(t))$ being a Markov process of economic factors, $(B(t))$ and $\tilde{N}$ standing for Brownian motion and compensated Poisson process respectively. We invest in assets using impulse strategies (τ_n, ξ_n) consisting of an increasing sequence of random times (τ_n) $(\tau_n \to \infty)$ at which we change the portfolio and ξ_n representing a vector number of assets in our portfolio after transaction. In the case of long time horizon we additionally impose an obligatory transaction to keep a bounded away from zero portion of the wealth process invested in each asset, since otherwise we would have trivial nondiversified portfolio. The class of impulse strategies form a feasible approximation class of an optimal control or even contains an optimal control depending on the form of the costs - gain functional. In particular, in the case when we have fixed plus proportional transaction costs or we assume decision time lag after each transaction the optimal control naturally belongs to this class. The value function corresponding to impulse control problems is a solution to certain quasi variational inequalities, which can be approximated by a sequence of stopping problems (variational inequalities), each of them we solve with the use of the penalty method. In the talk several results concerning growth optimal and risk sensitive growth optimal portfolios with fixed plus proportional transaction costs and simply proportional transaction costs will be presented. It will be shown in particular, that growth optimal impulse control for proportional transaction costs is also optimal for fixed plus proportional transaction costs.