Session proposition
Factorization of boundary value problems and applications

March 27, 2009

In this session some aspects and applications of the factorization of linear elliptic boundary value problems are presented. This method is inspired by optimal control theory. We use here spatially the technique of invariant embedding which is used in time to compute optimal feedback in control. In a basic presentation the method deals with a problem set in a cylindrical domain. In the symmetric case we consider the state equation as the optimality system of a control problem, one space variable playing the role of time. The problem is embedded in a family of similar problems defined over subdomains of the initial domain. It allows to decouple the optimality system as for the derivation of the optimal feedback. So one can factorize a second order elliptic boundary value problem in two first order Cauchy problems of parabolic type. These problems are decoupled : one can solve one problem in one space direction ("descent phase") then the other problem in the opposite direction ("climbing phase"). This decoupling technique also works in the nonsymmetric case.

- A.M. Ramos: *Presentation of the factorization method in a simple situation: study of the Riccati equation.*
  This talk is intended as an introduction to the session. It will present the general idea of the method in a basic situation but it will also stress the mathematical difficulty in deriving the particular Riccati equation of the problem.

- J. Henry, F. Jday: *Extension of the factorization method to non-cylindrical domains.*
  We consider here an extension of the cylinder where the section evolves smoothly along the axis. In the spatial invariant embedding, the subdomains are limited by these variable sections. Two approaches will be presented. Either one can consider a derivation field parallel to the axis of the "pseudo cylinder". Then the Riccati equation is unchanged in the pseudo cylinder but boundary conditions appear on the lateral boundary of the domain. This case is treated with the fictitious domain method. Or one uses Zolesio's velocity technique to describe the variation of the section. Then a new term appears in the Riccati equation.

- A. Ben Abda, J. Henry, F. Jday: *Reconstruction of missing boundary data using the factorization method.*
  The data completion problems consists in estimating missing boundary data on a part of the boundary with the use of extra measurements on another part of the boundary. This ill posed problem is set as an optimal control problem with a Tychonov

regularization. The factorization method is used simultaneously on the state and adjoint equations. Once the corresponding Riccati equations are solved, for each new set of data the estimate of the missing data is obtained by solving a parabolic problem depending on the Riccati operators and solving a linear system.

- A.M. Ramos, B. Louro, *Asymptotic expansion of Dirichlet to Neumann operators on slim bodies.*
  The boundary value problems factorization method furnishes the Dirichlet to Neumann operator on a part of the boundary. In this talk slim bodies are considered and an asymptotic expansion of the factorized version of the boundary value problem is presented. This yields an asymptotic expansion of the Dirichlet to Neumann operator and the equations satisfied by each term of the expansion.

- B. Louro, M. Orey: *The QR factorization of boundary value problems.*
  In this talk we consider linear elliptic boundary value problems with overdetermined boundary values. The solution of the problem is then defined in the least square sense. We apply the factorization method to the normal equation of the least square problem. As the factorization method can be viewed as an infinite dimension extension of the Gauss LU factorization, in this case the factorization can be viewed as an extension of the QR method.