Sample average approximation for chance constrained programming with applications

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Chance constrained programming, or probabilistic programming, was introduced by Charnes, Cooper and Symmonds in 1958 [CCS] and has been extensively studied since. Unlike two-stage programs with recourse, chance constrained programming is a qualitative approach in which the decision maker is interested in satisfying his goal most of the time, that is, he admits constraint violation for some realizations of the random events.

Although chance constraints were introduced more than 50 years ago, little progress was made until recently. One of the difficulties is that chance constraints usually give rise to non-convex problems. Furthermore, even checking if a given candidate is feasible is a difficult task and one usually needs to employ Monte-Carlo simulation.

We study sample approximations of chance constrained problems through the sample average approximation (SAA) approach and state the related convergence properties. We discuss how to use SAA to obtain good candidate solutions and bounds for the optimal value of the original problem. The proofs of the results and numerical examples can be found in [PAS].

We apply SAA to an actuarial problem consisting in the determination of the minimum provision an economic agent must have in order to meet a series of future payment obligations (the *hurdles*) with sufficiently high probability. The problem is a generalization of the hurdle race-problem, proposed by Vanduffel et.al. [VDGK]: we assume that all payment obligations are taken jointly. Further extensions such as having stochastic hurdles can be handled by SAA at almost no extra cost.

References

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