

# **Optimal Control of an $M/M/s$ Make-to-Stock Queue with Multiple Customer Classes and Lost Sales**

Mehmet Murat Fadiloğlu, Önder Bulut

*Department of Industrial Engineering, Bilkent University, Ankara, Turkey*

Contact E-mail: mmurat@bilkent.edu.tr

We consider the problem of production and stock rationing control in a make-to-stock production system with lost sales, multiple exponential servers --production channels-- and several customer classes that generate independent stationary Poisson demands. At decision epochs, in conjunction with the rationing decision, the control specifies whether to increase the number of operational servers or not. Previously placed production orders cannot be cancelled. We model the system as an  $M/M/s$  make-to-stock queue and characterize properties of the optimal cost function, and the optimal production and rationing policies. We show that the optimal production policy is a state-dependent base-stock policy, and the optimal rationing policy is of threshold type. Furthermore, we prove that the rationing levels are nonincreasing in the number of operational channels. We quantify the additional gain that the optimal policy provides over the --suboptimal--base-stock policy proposed in the literature, along with the value of full flexibility of canceling the production orders.

The optimization model utilized in this study is a continuous-time Markov decision process with infinite-horizon-discounted-cost criteria. The state space for the model is two-dimensional. The first dimension designates the inventory on-hand, while the second dimension keeps track of the number of the operational servers. Yet, since the decisions affect the second dimension in an immediate fashion, we define it as the number of the operational servers at the time of the previous customer arrival or production completion. This subtle definition allows us to evade problems in converting the model to a discrete-time Markov decision process via uniformization. Once the equivalent discrete-time process is obtained, its structural properties are proven using the operator technique.

This work constitutes a significant extension of the literature in the area of control of make-to-stock queues, which considers only a single server. We allow an arbitrary

number of servers in our model. As the number of available servers increases, the optimal policy stops changing beyond a certain number of servers. Therefore, our work also handles the case of infinitely-many servers, i.e., exogenous supply system.