Approximation of the Probability Density Function of the Solution of an Optimization Process Considering Uncertainties

R.H. Lopez*, E. S. Cursi
Institut National des Sciences Appliqués (INSA) de Rouen
76860 Saint-Etienne du Rouvray CEDEX, France
*rafaelholdorf@gmail.com
eduardo.souza@insa-rouen.fr

Keywords: optimizaton, uncertainties, non-convexity, Polynomial Chaos Expansion.

This paper aims at approximating the probability density function (PDF) of the solution of an optimization process. It may be defined as full characterization, once the PDF of the optimum point is obtained. Thus, we have problems of the type

$$x^* = Arg\,Min\ F(x,b) \tag{1}$$

where F(…) is a function that depends on the independent variable *x* and on the random variable *b* whose PDF is assumed to be known. Once the function F(…) depends on random parameters, its optimum point *x** also does. Hence, we seek to approximate the PDF of *x**.

Thus, in this paper, a numerical method based on the Polynomial Chaos Expansion (PCE) [1] is proposed to pursue the approximation of the PDF of the optimal solution. Here, *x* in Equation 1 is expanded and it becomes

$$x(Z) = \sum_{i=0}^{\infty} Z_i \cdot \psi_i\left(\{\xi\}_{n=1}^{\infty}\right) \tag{2}$$

where $\psi_i\left(\{\xi\}_{n=1}^{\infty}\right)$ are functions that depend on the random variables ξ whose PDF is also known and Z_i are deterministic coefficients to be determined. Now, the new problem is to find the optimum deterministic coefficients Z^* of the PCE and, as the functional to be minimized depends on random variables (i.e., *b* and ξ), we aim at minimizing one of its statistical characteristics (i.e., mean).

To pursue the minimization of such functional, stochastic approximation techniques have been employed, such as the method of the stochastic quasi-gradient [2]. Then, this method was coupled with the random perturbation of the gradient algorithm to optimize non-convex functions [3].

Convex, non-convex functions (i.e. Goldstein-Price, Shekel, Rosenbrock) as well as a structural mechanical problem were analyzed using different types of random parameters.

Then, the proposed method was validated comparing its results to those obtained through Monte Carlo Simulations.

References

[1] R. Ghanem and P. Spanos, “Stochastic finite elements: a spectral approach”, Springer-Verlag, New York, USA, 2003.

[2] R. Wets, "Stochastic approximation", in “Handbook for operations research and management sciences”, Elsevier Science Publishers, vol. 1, pp. 573 - 629, 1989.

[3] J.E. Cursi and M.B.S. Cortes, “General Genetic algorithms and simulated annealing perturbation of the gradient method with a fixed parameter. In: “Topping, B.H.V. (Ed.), Developments in Neural Networks and Evolutionary Computing for Civil and Structural Engineering”, pp. 189-198, 1995.