An elastic demand transit assignment model for congested transit networks

by

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The development of user equilibrium models of increasing complexity for passenger transit assignment on public transportation services has been driven by the need of using these models in design studies, capable to reflect congestion effects on the transportation system. The always increasing need for transportation in densely populated urban areas on the one hand, and the policies for increasing the use of public transportation services due to the requirements for environmental sustainability on the other, depict a horizon in which the modelling of congestion effects must be taken into account mandatorily. This also applies in transportation planning studies as well, on which the user equilibrium assignment of passengers plays a central role.

The paper of Chriqui and Robillard (1975) introduced the notion that passengers can select a subset of attractive lines and board the first vehicle arriving at a stop in order to minimize the expected sum of travel times plus waiting times at stops. An extension to this idea was carried out by Spiess (1984) by means of the concept of strategy. The resulting model was shown to be equivalent to a linear programming problem on a multidestination network. A further extension including partially the effects of congestion is done in Spiess and Florian (1989) by associating discomfort functions to transit line segments, so that the resulting equilibrium model can be formulated as a convex programming problem. In all these models line frequencies are assumed to be constant and not affected by the limited capacity of vehicles, which may cause that passengers may not board on the first attractive vehicle since their arrival at the stop. This implies that waiting times at stops may be underestimated by solely considering line frequencies, because a queuing for the first vehicle with available capacity may arise if the arrival flows of passengers at stops are high. In this case, perceived frequencies by passengers are smaller than line frequencies.

In the paper by Cepeda *et al* (2006) a transit user equilibrium assignment model consistent with the concept of strategies is developed allowing a full representation of the congestion effects due to the decrease of the frequencies experienced by passengers at transit stops. The computation of user equilibrium is carried out by the minimization of a nondifferentiable gap function, carried out heuristically by means of the method of successive averages. In Codina (2009) a reformulation of the transit equilibrium assignment model of Cepeda *et al.* is done showing that it can be expressed as an equivalent variational inequality problem (V.I. in the following). The algorithmic potentials of this formulation are explored as well as a class of approximations.

Along with the congestion effects, the multimodality in many transportation systems has led also to some authors to formulate elastic demand transit assignment models (Lei and Chen (2004)) in order to use them in problems of design of public transit lines as in Lee, Y-J. and Vuchic, V.R. (2005), but none of these models are based on the concept of strategy, or are consistent with it.

This paper is focused on the extensions that the reformulation of the model of Cepeda *et al*. as a V.I. problem permits. After briefly reviewing the most significant transit assignment models, the reformulation of the congested transit assignment model of Cepeda *et al.* (2006) made in Codina (2009) is discussed. Next it is shown how an elastic demand model can be formulated also as a V.I. problem This extension plays the same role than the work of LeBlanc and Farhangian (1981) for the case of the computation of the user equilibrium in elastic demand traffic assignment models. The case of multinomial logit and hierarchical logit demand models are introduced and analyzed.

In order to solve the elastic demand model formulated, first the heuristic application of the method of successive averages is shown. Next, the transformation of Gartner is applied in order to reduce the model to the inelastic case. Finally, an application of the generalized Benders decomposition algorithm of Fuller and Chung (2005) for V.I.'s is shown. It must be remarked at this point that the V.I. problem formulated does not present in the general case monotonicity properties so as to guarantee a convergence of the algorithm. However, it must be remarked that the under very mild conditions, which are indeed exhibited by the models examined in these extensions, the method of Fuller and Chung makes a gap function to decrease monotonically. By means of a set of computational experiences on medium size networks it is shown the viability of the proposed models and methods. The effects of the congestion are examined using for the passenger's waiting times at stops the theoretical queuing models developed in Cominetti and Correa (2001).

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