Experiments with two-row cuts

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Abstract

In the last 15 years, generic cutting plane techniques have played a major role in the progress of mixed integer linear programming (MILP) solvers. Most of the cutting plane algorithms implemented in current state of the art solvers rely on cuts that can be derived from a single equation (such as Gomory mixed integer cuts, MIR cuts, lift-and-project cuts, lifted cover inequalities). A natural idea to build more efficient cutting plane algorithms is to use cuts which need more than one equation to be derived.

Early work on the Corner Polyhedra pioneered by Gomory and Johnson [8, 7, 9] provided methods for studying inequalities arising from sets composed of multiple rows. In recent year there has been a renewed interest in this direction of research. In particular, several authors have studied from a theoretical point of view cuts which can be derived from two equations[1, 2, 5, 4]. Espinoza gave computational evidence that cuts derived from two equations can effectively improve MIP solver performance in some cases.

In a recent work [3], we addressed the problem of comparing the strength of cuts which can be derived from a single equation to that of cuts which are derived from two equations from a theoretical point of view. In particular, there exist examples of integer programs with two constraints where all the integrality gap can be closed by a single inequality derived from two rows while the value of the relaxation obtained by adding all inequalities derived from one equation (where the equation may be obtained from a combination of the constraints of the problem) can be arbitrarily close to 0. This example also suggests that some particular inequalities may be useful in a more general context

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We propose here to study from a computational point of view the applicability of cuts derived from two equations based on our theoretical study. After having recalled the different types of cuts which may be obtained from two equations and the examples where cuts obtained from two equations are useful, we will present how to apply the results in a general framework. In particular, we will study how coefficients of the cuts for integer variables can be strengthened using the results of Dey and Wolsey[5], we will discuss several cut selection techniques and we will present preliminary computational result to assess the strength of the various methods proposed.

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