Stabilisation of the second order system with a time delay controller

Jerzy Baranowski and Wojciech Mitkowski

Department of Automatics, Akademia Górniczo-Hutnicza, Kraków, Poland

1 Introduction

The work considers the problem of second order system with a time delay controller $u(t) = Kx(t-h)$. In such situation, the closed loop system is an infinitely dimensional one (with a discrete spectrum). In this abstract only a simple example is presented, for which the area of asymptotic stability in the parameter space $(K \times h)$ was determined. Similar effects associated with the influence of delays $h > 0$ were observed for higher order systems, stabilised with time delay controller (Chrapała, 2005; Niculescu and Abdallah, 2000).

2 Second order system

Let us consider a linear, second order, ordinary differential equation

$$\ddot{x}(t) + x(t) = u(t) \tag{1}$$

where $x(t) \in \mathbb{R},\ u(t) \in \mathbb{R}, t \in \mathbb{R}$. Equations such as (1) can be used for example as a model of a LC circuit. This system is characterised with undamped oscillations.

3 Stabilising controller

For (1) one can use for example the following control law

$$u(t) = K\dot{x}(t) \tag{2}$$

with $K < 0$. For such K the closed loop system (1)-(2) is asymptotically stable (oscillations vanish). The controller gain K represents the "acting force" of the controller however the "velocity" $\dot{x}(t)$ which represents the direction of changes at the current moment, is generally difficult to obtain from the engineering standpoint. To determine $\dot{x}(t)$ one needs past information or "good intuition".

Let $h > 0$ be a parameter that includes the past (in the language of control theory $h > 0$ is a time delay (see i.e. Górecki, 1971)). Let us consider a following control law

$$u(t) = Kx(t-h) \tag{3}$$

In this case, system consisting of (1) and (3) is described by a delayed differential equation

$$\ddot{x}(t) + x(t) - Kx(t-h) = 0 \tag{4}$$

The area of asymptotic stability (see. Abdallah et al., 1993; Mitkowski and Skruch, 2005; Niculescu and Abdallah, 2000) for system (4) depends on two parameters K and h. It is determined by following inequalities for $n = 0,\ 1,\ 2,\ 3,\ \ldots$

$$\begin{aligned} 0 < K < \frac{1+4n}{1+4n+8n^2} \\ \frac{2n\pi}{\sqrt{1-K}} < h < \frac{(2n+1)\pi}{\sqrt{1-K}} \end{aligned} \tag{5}$$

4 Conclusion

In the full version of the paper, more general class of systems will be considered, that is:

$$\ddot{x}(t) + Ax(t) = Bu(t) \tag{6}$$

$$y(t) = Cx(t) \tag{7}$$

where $x(t) \in \mathbb{R}^n$ and A, B, C are real matrices of appropriate dimensions. Systems of this type were considered among the others by (Skruch, 2005) and can be used for example to model an LC chain circuits (Mitkowski, 2004).

Also we will consider control laws in the form of delayed feedback

$$u(t) = Kx(t-h) \tag{8}$$

which with $K = K_1C$ can be considered an output feedback.

Moreover it should be noted, that for K close to 0 (but positive) system (4) is asymptotically stable for

$$h \in (2n\pi, (2n+1)\pi), n = 0,\ 1,\ 2,\ 3,\ \dots \tag{9}$$

but for $h \in ((2n+1)\pi, 2n+2)\pi)$ system (4) is unstable. It can be seen then, that using the past information, with h chosen according to (9) we can stabilise the present occurences. With larger K - meaning more "powerfull" controller, the choice of possible delays reduces.

That is why the choice of appropriate parameters for the controller (3) is crucial even in such simple systems as (1). In the full version of the paper, an algorithm for finding the optimal parameters of considered feedback for system (6), in the respect to chosen performance index will be presented and discussed. This algorithm will be based on sequences of finite dimensional approximations of the controller. These approximations will include Pade approximations in the domain of transfer functions and other in time domain. The following list of references only includes key elements and will be substantially expanded.

References

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