

The Windy Clustered Prize-collecting Arc Routing Problem.

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Prize-collecting Arc Routing Problems are arc routing problems where, in addition to the cost function, there is a profit function on the edges that must be taken into account only the first time that an edge is traversed. In the Clustered Prize-collecting Arc Routing Problem there are clusters of arcs and, for each cluster, it is required that either all its links are serviced or none of them is serviced. This problem have been studied in our paper [1]

This work presents this problem and its extension to the Windy Clustered Prize-collecting Arc Routing Problem and studies properties and dominance conditions that allow the formulation of the problem to be strengthened. Also an exact algorithm for optimally solving the problem is proposed. At the root node of the enumeration tree the algorithm generates upper and lower bounds obtained from solving an iterative LP-based algorithm in which violated cuts are generated when possible. A simple heuristic that generates feasible solutions provides lower bounds at each iteration. The numerical results from a series of preliminary computational experiments with various types of instances assess the good behavior of the algorithm.

Prize-collecting Arc Routing Problems (PARPs) are arc routing problems where, in addition to the cost function, there is a profit function on the edges that must be taken into account at most once: only when an edge is serviced. PARPs were defined in Aráoz, Fernández and Zoltan [3], and the so called Privatized Rural Postman Problem (PRPP) has also been studied by Aráoz, Fernández and Meza in [2]. Like in other arc routing problems in the PARPs we assume that the demand of service is placed at the edges of a graph. However, as opposed to typical arc routing problems, there is no specific arc subset to be traversed. Instead, we assume that giving service to an edge will incur not only a cost (associated with displacement), but also a profit (associated with servicing edges). Like in other arc routing problems, each arc incurs a cost every time it is traversed. On the contrary, the profit of each edge serviced in the route will be collected only once, independently of how many times the edge is traversed. In the PARP we look for traversals that maximize the total servicing profit minus the displacement cost.

As shown in [3] the Rural Postman Problem (RPP) is a particular case of PARP. In fact, PARPs constitute a generalization of most Edge Routing Problems with one single route.

The PARP can be seen as the Edge Routing version of the Travelling Salesman Problem with Profits (otherwise known as the Profitable Tour Problem-PTP [5]). Using a transformation similar to that of Lenstra and Rinnooy Kan [6] for transforming a TSP into a Rural Postman Problem instance, we can also transform the PTP into a PARP.

In this work we present the **Clustered Prize-collecting Arc Routing Problem (CPARP)** that we solved in [1]. CPARP is a PARP where, in addition, we consider the components (clusters) defined by the edges with demand, and for each cluster we require that either all its links are serviced or no link of the cluster is serviced, however, it is not necessary to service all edges of a

component before moving on to another one if this results in a better solution. Here we present new results for CPARP and the work done in the **Windy Clustered Prize-collecting Arc Routing Problem (WCPARP)**

The difference is than in Windy problem the cost of traversing an arc could be different according to the direction of the transversal.

Potential applications of CPARP and WCPARP include, among others, determining the service for garbage collection or street cleaning in districts, neighborhoods in a given area. Indeed, while it is not acceptable that only a part of a given neighborhood is serviced, the whole neighborhood might not be profitable for the servicing company. As we will see, when we pose the additional requirement that either all the links of a cluster are serviced or no link of the cluster is serviced, we can prove some dominance conditions that otherwise do not hold. These properties allow some transformation of the original graph that results in a stronger polyhedral formulation of the problem. This formulation has an exponential number of inequalities of two types. Inequalities that guarantee the connectivity of the traversal with the depot, and inequalities that guarantee the even degree of the nodes. For both CPARP and WCPARP we develop binary variable models, which allows a particular case of the so called co-circuit inequalities of Barahona and Grötschel [4] to assure even degree of the nodes. For the WCPARP we also considered the natural integer variable approach.

We propose an exact branch-and-cut algorithm to optimally solve the problem. At the root node we obtain an upper bound by solving the LP relaxation of the model. Due to the exponential number of constraints we use an LP based iterative scheme that starts with a small number of inequalities and at each iteration reinforces the current model with violated inequalities. This is possible since, as we will see, we can solve exactly the separation problem in polynomial time for the two exponential families of inequalities. We also propose a simple heuristic to generate feasible solutions that provides lower bounds at each iteration. For analyzing the performance of the proposed algorithm we have run a series of computational experiments with a set of benchmark instances used for other arc routing problems in the literature.

Finally we present preliminary numerical results.

References

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