NUMERICAL SOLUTION OF A MINIMAX OPTIMAL CONTROL PROBLEM USING THE PONTRYAGIN MAXIMUM PRINCIPLE

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Keywords: Minimax problems, Pontryagin maximum principle, numerical solution, optimal control, convex analysis.

Abstract

We consider in the interval $[0,T]$ a dynamic system which evolves according to the ordinary differential equation

$$\left| \begin{array}{ll} \dfrac{dy}{ds}(s) = g(y(s),\alpha(s)) & 0 \le s \le T, \\ y(0) = x \in \Omega \subseteq \mathbb{R}^r, & \Omega \text{ an open domain.} \end{array} \right. \tag{1}$$

The optimal control problem consists in minimizing the functional J

$$J : (x,\alpha(\cdot)) \in \Omega \times \mathcal{U} \mapsto \operatorname{ess\,sup} \{ f(y(s),\alpha(s)) : s \in [0,T) \} . \tag{2}$$

The set of controls is

$$\mathcal{U} = \{ \alpha : [0,T] \to A \subset \mathbb{R}^m : \alpha(\cdot) \text{ measurable} \}$$

and the set of controls A is compact.

This problem arises, for example, when we want to minimize the maximum deviation of the controlled trajectories with respect to a given special trajectory. This differs from those problems usually considered in the optimal control literature, where an accumulated cost is minimized. As considering an accumulated cost is not always the best method to qualify a controlled system with an unique real parameter, problems of this type has received considerable interest (see e.g. Barron and Ishii (1989), Barron E.N. (1990), and Barles G. et al. (1994)).

The objective of this work is to approximate in a numerical way an optimal control policy $\hat{\alpha}$ such that:

$$J(x,\, \hat{\alpha}(\cdot)) = u(x) := \inf \{ J(x,\alpha(\cdot)) : \alpha \in \mathcal{U} \} . \tag{3}$$

Hypothesis. We assume that f, g are bounded and uniformly continuous functions on $\Omega \times A$ and there exists $\partial g / \partial x$ which is a bounded and continuous function also on $\Omega \times A$.

We present here a simplified version of the problem, where we suppose that

$$\left| \begin{array}{l} g(y,\alpha) = g_1(y) + g_2(y)\,\alpha \\ A \text{ is convex} \\ f \text{ is independent of } \alpha. \end{array} \right. \tag{4}$$

We also suppose that the trajectory $y(\cdot)$ remains in Ω, for any control belonging to $\mathcal{U}$.

Results. The principal results obtained in our work are the following ones:

- We obtain a set of necessary conditions that must be verified by any optimal control policy.
- These necessary conditions are given in terms of Lagrange multipliers (adjoint vectors), as it is usual in Pontryagin's approach.
- We define an approximation in discrete time of the original problem.
- The solutions of the approximated problem converge to the continuous solution with a rate h.
- We present a computational algorithm to obtain the solution of the discrete problem and numerical results.

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