An Active-Set Strategy for Linearly Constrained Optimization *

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A method for linearly constrained optimization which modifies and generalizes recent box-constraint optimization algorithms is introduced. The new algorithm is based on a relaxed form of Spectral Projected Gradient iterations. Intercalated with these projected steps, internal iterations restricted to faces of the polytope are performed, which enhance the efficiency of the algorithms. Convergence proofs are given and numerical experiments are included and commented. Software supporting this paper is available through the Tango Project web page: *http://www.ime.usp.br/∼egbirgin/tango/*.

The Spectral Projected Gradient (SPG) method was introduced, analyzed and implemented in [7, 8, 9]. This method combines the basic spectral-step ideas [3, 12, 13] with projected gradient strategies [4, 10, 11]. The extension of the spectral gradient method to smooth convex programming problems [1, 6, 5, 7, 8, 9] was motivated by the surprisingly good performance of the spectral gradient for large-scale unconstrained minimization [13]. The SPG method is applicable to convex constrained problems in which the projection onto the feasible set is easy to compute. The main drawback for the application of SPG to arbitrary convex (in particular, linearly constrained) problems is the expensiveness of computing projections in general cases. In the present research, in order to apply SPG ideas to general linearly constrained optimization, we used two different strategies:

1. Enlarging the set onto which projections must be computed.
2. Combining the SPG-like iterations with easy-to-compute "essentially unconstrained" iterations which provide practical efficiency to the method.

The enlargement of the projection set involves elimination of constraints. So, by means of this procedure, projections become less expensive. Radical elimination would involve preserving only active constraints in the projection set, but this elimination would be inefficient and even convergence proofs would be impossible. A careful management of the constraints that must be preserved at each state of the calculation gives rise to the Partial Spectral Projected Gradient (PSPG) strategy introduced in the present paper.

The strategy of combining SPG-like iterations with iterations restricted to proper faces was already used in [2, 5, 6] in connection to box-constrained optimization. The main ideas of the present research are inherited from the box-constrained ideas of [2, 6] but the consideration of

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general linear constraints, instead of merely bounds, imposes different algorithmic decisions. In the case of boxes, projections are trivially computed, therefore it is inexpensive to test, at every iteration, if the face where the current approximation lies must be preserved or not, using the comparison of internal and external components of the constrained steepest descent direction [6]. In the case of linear constraints we prefer to preserve the same active set until the boundary is reached or an approximate active-face constrained stationary point occurs. In this way we reduce, as much as possible, projection steps. The idea of combining PSPG steps with internal iterations can also be understood from the point of view of active-set methods.

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