Interior point method applied to the AC active-reactive optimal power flow problem using nonlinear corrections in all optimality conditions

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Introduction

The predictor corrector interior point methods is developed to the AC active and reactive optimal power flow problem. The representation of the tensions through Cartesian coordinates is adopted, once the Hessian is constant and the Taylor expansion is accurate for the second order term [4]. The advantage of working with polar coordinates, that easily model the tension magnitudes, lose importance due to the efficient treatment of inequalities proportioned by the interior point methods, in particular on power systems problems [2, 3]. As a consequence of avoiding polar coordinates, all constraints of the obtained model are quadratic allowing the application of non-linear corrections in all optimality conditions as oppose as the traditional approach where the corrections are applied only in the complementary conditions. Observe that these proposed corrections are not needed for linear programming problems [5] since the constraints are linear and can not of be obtained in a closed form for many nonlinear programming problems such as the AC active and reactive optimal power flow problem using polar coordinates.

In a nutshell the corrections of quadratic constraints can be obtained as follows. Given a constraint in the form $XAx - b = 0$, where A is a constant matrix of appropriated dimension, b is a column vector also constant and x represents the problem variables. Finally, we use the standard notation in interior point methods to represent diagonal matrices were the diagonal entries are the entries of a vector: $X = diag(x)$.

Applying Newton's method to these constraints gives rise to:

$$(XA + diag(Ax))\Delta x = b - XAx$$

After computing Δx the new point would be $\tilde{x} = x + \Delta x$. The constraint is

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then given by:

$$\begin{aligned}\tilde{X}A\tilde{x} - b &= (X + \Delta X)A(x + \Delta x) - b \\ &= XAx + XA\Delta x + \Delta XAx + \Delta XA\Delta x - b \\ &= XAx + \Delta XA\Delta x + (b - XA\Delta x) - b \\ &= \Delta XA\Delta x.\end{aligned}$$

Therefore the nonlinear correction of $XAx - b = 0$ is given by $\Delta XA\Delta x$.

The predictor corrector interior point method is then applied to the AC optimal power flow problem using Cartesian coordinates optimality constraints. In order to obtain a primal-dual method, the slack dual variables are defined as in [1], giving raise to the complementary conditions.

Before the application of the method, the number of variables of the problem is reduced through the elimination of free dual variables. This elimination does not modify the sparse pattern of the problem. The linear system obtained can be further reduced to the dimension of twice the number of buses also with minor changes in the sparse structure of the matrices involved. Moreover, the final matrix is symmetric reducing the computational effort per iteration still further.

Computational experiments for IEEE and Brazilian systems are presented showing the advantages of the proposed approach.

References

[1] A. S. El-Bakry, R. A. Tapia, T. Tsuchiya, and Y. Zhang, *On the formulation and the theory of the Newton interior-point method for nonlinear programming*, Journal of Optimization Theory and Applications, 89 (1996), pp. 507–541.

[2] J. A. Momoh, M. E. El-Hawary, and R. Adapa, *A review of selected optimal power flow literature to 1993, part II Newton, linear programming and interior point methods*, IEEE Transactions on Power Systems, 14 (1999), pp. 105–111.

[3] V. H. Quintana, G. L. Torres, and J. Medina-Palomo, *Interior point methods and their applications to power systems: A classification of publications and software codes*, IEEE Transactions on Power Systems, 15 (2000), pp. 170–176.

[4] G. L. Torres and V. H. Quintana, *An interior point method for nonlinear optimal power flow using voltage rectangular coordinates*, IEEE Transactions on Power Systems, 13 (1998), pp. 1211–1218.

[5] S. J. Wright, *Primal–Dual Interior–Point Methods*, SIAM Publications, SIAM, Philadelphia, PA, USA, 1996.