ESTIMATION OF THE DYNAMICS OF ENERGETIC PRICES USING ONE FACTOR MEAN REVERTING MODELS

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ABSTRACT

The implementation of Mean Reverting Models has been frequently used to model Commodities in the energetic market [1] and [2], such as crude oil, natural gas, and electricity spot prices. The use of such processes supposes the development of mathematical models immerse in the context of stochastic calculus and/or time series. The stochastic differential equations most used to model this behaviour are based on the Pillipovic and Hull & White models of one factor, two factors with or without correlation [1] or three factors [2].

Many of the characteristics of these processes can be analysed from the simulated processes in which the parameter present in the model can be imposed. A really important (interesting) problem from the point of view of the real applications is to determine the parameters from the historical information of the asset, in other words, from the past observations of the process to be modeled.

The estimation of parameters is a common process in modelling systems thought it is not simple. In the present work a particular interest in the estimation process is recognized because both the future values simulation and the option pricing are procedures that depend on the estimated parameters found by any of them.

There are several methods for parameter estimation and in general to obtain unobservable information from a series of data. For the general case of parameter estimation in diffusion processes some methodologies have been used: Kalman filters in their linear version (especially for the case in which the noise is additive), minimum variance estimators [1]; besides the possibility of doing such estimates through methods like *Maximum likelihood* , moments method, infinitesimal generator and other methods based on simulation [3]. Most of these methods have complicated representations, moreover, many of the important characteristics of the estimated parameters cannot be determined analytically, for instance, the bias, the consistency, etc. (which is the case of estimations based on simulation)

For the reason raised above, it is presented in this work a methodology for the parameters estimation in Mean Reverting Models using the *Maximum likelihood* method, subject to the random Euler scheme. The proper characteristics of *good estimators* are derived form this parameters estimation technique.

Although the case of one factor model is presented at first, these ideas can be extrapolated to more general models. In [1] a careful study of general two factors models is presented based on these ideas.

The one factor model for a mean-reverting process has the form

$$dS_t = \alpha\left(\mu - S_t\right)dt + \sigma\, S_t^{\gamma}\, dB_t\,; \quad \gamma = 0{,}1\ , \tag{1}$$

where α, μ y σ are constants and $\{B_t\}_{t\geq 0}$ is a standard Brownian motion. For the case in which the noise is additive $\gamma = 0$ it is easy to obtain an explicit solution for (1) and from it to find the maximum likelihood estimators. This is the same model in which the no linearity makes it impossible to use the estimation process with Kalman filters in its traditional form.

Because of the reason previously mentioned, it is considered the model (2), the discretization of the stochastic differential equation (1) using the random Euler scheme that is just a “generalization” of the deterministic case. This approach has great characteristics for the diffusion processes since the conditions under which the solution obtained *converges* to the analytical solution can be determined [4].

$$S_{t_i} - S_{t_{i-1}} = \alpha\left(\mu - S_{t_{i-1}}\right)\Delta_i t + \sigma\, S_{t_{i-1}}^{\gamma}\, \Delta B_{t_i}\,; \quad \gamma = 0{,}1\ , \tag{2}$$

Furthermore, this scheme allows obtaining maximum likelihood estimators in a simple way, in such a way that the parameters obtained by this procedure should converge to the “*real parameters*”. The manner of achieving such estimation is based on the definition the new variable:

$$X_i = \left[S_{t_i} - S_{t_{i-1}} - \alpha\left(\mu - S_{t_{i-1}}\right)\Delta_i t\right]\left[S_{t_{i-1}}^{\gamma}\right]^{-1} = \sigma\, \Delta B_{t_i}\,; \quad \gamma = 0{,}1, \tag{3}$$

So that $X_i \sim N(0, \sigma^2 \Delta t)$. This reviews the complete development for the case of the one factor model with constant parameters.

The equation (4) presents the one factor model in which the long term mean $\mu(t)$ is a deterministic function of time (unknown), the parameters α y σ are constant and $\{B_t\}_{t\geq 0}$ is a standard Brownian motion. The estimation strategy is very similar to the one previously described, yet for this case it is necessary to construct punctual estimators for the constant parameters and a numeric estimation for the long term mean.

$$dS_t = \alpha\left(\mu(t) - S_t\right)dt + \sigma\, S_t^{\gamma}\, dB_t\,; \quad \gamma = 0{,}1\ , \tag{4}$$

In [1] an alternative way of estimation is presented using a very similar random Euler scheme showed in (2) subject to a definition of a new variable

$$X_i = \left(S_i - S_{i-1} - \left[\alpha\left(\varphi_{i-1} - S_{i-1}\right) + \dot{\varphi}_{i-1}\right]\Delta t\right)\left(S_{i-1}^{r}\right)^{-1} = \sigma \cdot \Delta B_i \sim N(0, \sigma^2 \Delta t)\,. \tag{5}$$

In this case it should be estimated $\varphi(t)$ as well, which represents the long term expected value for the solution process of the equation (1).

References

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