Extension of the factorization method to non-cylindrical domains.

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We consider an extension of the factorization method to domain that are deformation of cylinders where the section evolves smoothly along the axis. More precisely we consider the following problem

$$(\mathcal{P}_0)\begin{cases} -\Delta u = f & \text{dans } \Omega, f \in L^2(\Omega) \\ u|_\Sigma = 0, & -\dfrac{\partial u}{\partial n}|_{\Gamma_0} = u_0 \in (H^{1/2}_{0,0}(\mathcal{O}_0))' \quad u|_{\Gamma_a} = u_1 \in H^{1/2}_{0,0}(\mathcal{O}_a). \end{cases}$$

Ω is defined by its sections $\Omega = \displaystyle\bigcup_{x=0}^{a}(\{x\}, \mathcal{O}_x)$. The sections $\mathcal{O}_x$ are bounded sets in $\mathbb{R}^{n-1}$ depending regularly on x, without a change of topology. Let $\Sigma = \displaystyle\bigcup_{x=0}^{a}(\{x\}, \partial\mathcal{O}_x)$ be the lateral boundary of Ω. We denote by n the exterior normal to Ω on Σ and let ν be the exterior normal to $\mathcal{O}_x$ in the plane $\mathbb{R}^{n-1}$ at a point of $\partial\mathcal{O}_x$. The domain Ω is limited by the faces $\Gamma_0 = \{0\} \times \mathcal{O}_0$ and $\Gamma_a = \{a\} \times \mathcal{O}_a$ and the lateral boundary Σ.

In the spatial invariant embedding, the subdomains are limited by a variable section $\mathcal{O}_s$ at $x = s$. Two approaches will be presented. Either one can consider a derivation field parallel to the axis of the “pseudo cylinder”. Then the Riccati equation is the same in the pseudo cylinder as in the cylinder case but boundary conditions appear on the lateral boundary of the domain. Or one uses Zolesio’s velocity technique to describe the variation of the section. Then a new term appears in the Riccati equation.