Missing boundary data reconstruction the factorization method

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We consider in this work the following data completion problem for the Laplace equation in a cylindrical domain.

$$(P_0)\begin{cases} \triangle u = 0 & \text{in } \Omega \\ u = 0 & \text{on } \quad \Sigma \\ u = T & \text{on } \quad \Gamma_0 \\ \bigtriangledown u.n = \Phi & \text{sur } \quad \Gamma_0 \end{cases}$$

$\Omega =]0,a[\times O, O \subset \mathbb{R}^{n-1}$ (O is a smooth bounded open set and $a > 0$). $\Gamma_0 = \{0\} \times O$ and $\Gamma_a = \{a\} \times O$. The Neumann and Dirirchlet boundary conditions are given on Γ_0 while no condition is given on Γ_a. This problem has been known since Hadamard to be ill-posed. The problem is set as an optimal control problem with a regularized cost function. To obtain directly an approximation of the missing data on Γ_a we used the method of factorization for elliptic boundary value problems. This method allows to factorize a boundary value problem in the product of two parabolic problems. Here it is applied to the optimality system.

We apply to the method of invariant embedding developped by R. Bellman. In control theory this method gives the feedback optimal : One embedds the problem of control in a family of similair problems defined betewen the initial and the final time. In our work this invariant embeding is spacial : We embedds the problem (P_0) in a family of similar problems $(P_{s,k})$

defined on $\Omega_s =]0, s[\times O$. The factorisation method allows an explicit recovering of the boundary missing data when the Cauchy problem is rephrased into an optimal control one. In this formulation the Direchlet-Nemann and Neumann-Dirichlet operators do not depend on the data. The main feature of the proposed method is that if we have a collection of boundary data to recover, that is a collection of Cauchy data in Γ_0 one has only to solve the PDEś involving ω_1 and ω_2. Furthermore the proposed method permits to perform analytical computations which throw new insights into the missing data recovery problem.
The method presented here allows deep insight to the Cauchy problem for the cylinder in so far as it gives explicit formulation for the interfacial operators.