Semilinear Stochastic Equations with Fractional Brownian Motion

Bozenna Pasik-Duncan
University of Kansas

Empirical evidence suggests that the family of fractional Brownian motions is an important collection of processes for mathematical models of physical phenomena. Since some mathematical models are naturally formulated in a Hilbert space, e.g. stochastic partial differential equations and stochastic delay equations, it is important to consider the following type of semilinear stochastic equation in a separable Hilbert space V with a fractional Brownian motion

$$dX(t) = (AX(t) + F(X(t))dt + \Phi dB(t) \quad (1)$$
$$X(0) = X_0 \quad (2)$$

where A is the infinitesimal generator of an analytic semigroup, $X(t) \in V$ and $(B(t), t \geq 0)$ is a standard cylindrical fractional Brownian motion with $H \in (0,1)$ and H is the Hurst parameter that indexes the family of fractional Brownian motions. Weak solutions are considered for (1). A weak solution is obtained by an absolutely continuous transformation of measures from the measure for the solution of

$$dX(t) = AX(t)dt + \Phi dB(t) \quad (3)$$
$$X(0) = X_0 \quad (4)$$

to the measure for the solution of the semilinear equation (1). Sample path properties of (3) are also given because they are important for the verification of the absolute continuity of measures. Conditions are given for both the existence and the uniqueness of weak solutions of (1). The results dichotomize into the cases for $H \in (0, 1/2)$ and $H \in (1/2, 1)$ with the former having weaker assumptions than the latter. Some examples of stochastic partial differential equations that satisfy the assumptions for the results are given.