Discrete Optimization Model of Margining Investment Portfolios in Batches

Daniel Oron,* George Steiner,† Vadim G. Timkovsky*

March 29, 2009

Extended Abstract

We consider a discrete optimization model for the problem of margining customer accounts in batches which has wide applications in the financial services industry. The bookkeeping system of every brokerage firm automatically triggers this mandatory risk management operation at the end of every business day. After freezing all market prices, it produces account status slips before trades commence on the next business day. Batch margining is an important problem for large brokerage firms maintaining millions of accounts for their clients. Since they have only one night to process them, the speed of batch margining is crucial. Batch margining consists of the following two major parts with a complex precedence relationship: data retrieval from the database, including margining security positions, and margining the accounts that contain these positions. In the simplest form, the problem can be stated as follows.

The *margin shop* is a bipartite graph $(X \cup Y, B)$, which we call a *precedence graph*, with a vertex set $X \cup Y$, an edge set B and vertices $v \in X \cup Y$ representing *tasks* with processing times p_v. A connected component of the precedence graph is called a *job* in a margin shop, i.e., a job consists of tasks connected in the graph. An edge $xy \in B$ between $x \in X$ and $y \in Y$ determines the precedence relation between the tasks x and y, i.e., the completion time of x must not be later than the start time of y. Every task in X, Y must be processed by the machine M_X, M_Y, respectively.

The *margin-shop scheduling problem* is to find a nonpreemptive schedule of processing the tasks to minimize the *schedule length*. The set Y represents a *batch* of *margin accounts* that a brokerage firm maintains for its clients. Accounts in the batch have positions in financial securities of the set X. The precedence relation between the tasks x and y means that the account y has a position in the security x and the margin requirement for one unit of the security x must be calculated before the calculation of the margin requirement for the account y starts. This constraint follows from the fact

*Faculty of Economics and Business, The University of Sydney, Sydney, NSW 2006, Australia, {d.oron,v.timkovsky}@econ.usyd.edu.au.

†DeGroote School of Business, McMaster University, Hamilton, Ontario L8S 4L8, Canada, steiner@mcmaster.ca.

that any account-margining algorithm uses security positions' margins as part of the input. We assume that X is the union of the securities in the portfolios of the accounts in Y; therefore, each security x is contained in some account y and thus the precedence graph does not have isolated vertices.

Processing times p_x and p_y are margin calculation times for the security x and the account y, respectively. The machine M_X represents the server running database operations, including *stored procedures* for margining securities in X, and the machine M_Y represents the server for calculations outside the database. Jobs in a margin shop correspond to account batches that must be processed under specified sets of margin rule books/libraries which are usually not the same for different capital markets.

This paper reveals the relationship between the margin-shop scheduling problem, the flow-shop scheduling problem, the jump-number problem and the problem of finding a maximum red matching that is free of blue-red alternating cycles in a complete bipartite graph with blue and red edges; see Figure 1 for an example.

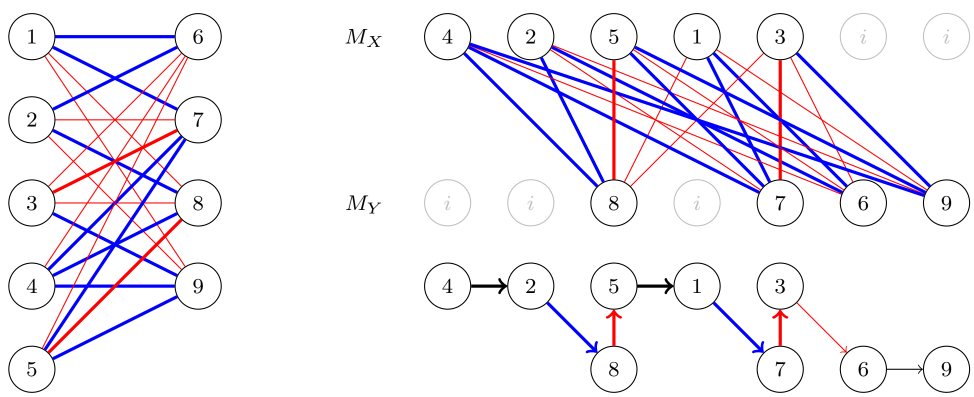

Figure 1: A unit-time margin-shop problem, where $X = \{1, 2, 3, 4, 5\}$, $Y = \{6, 7, 8, 9\}$. Edges of the bipartite graph $(X \cup Y, B)$ are blue. Edges of the complementary bipartite graph are red. The red matching $M = \{37, 58\}$ avoids blue-red alternating cycles. Circled is in the related schedule denote idle time units on M_X and M_Y. The last diagram represents the related linear extension L, where blue (28 and 17), thick black (42 and 51), thick red (85 and 73), thin red (36), thin black (69) arrows denote bumps (28 and 17), left-left (42 and 51), right-left (85 and 73), left-right (36), right-right (69) jumps, respectively.

We present the NP-hardness result for the unit-time margin shop in the case where the precedence graph is of degree four and efficient algorithms for solving the unit-time margin-shop problem in the case where the precedence graph is of degree at most two or a forest. Numerous attempts to establish the complexity status of the case where the precedence graph is of degree at most three were unsuccessful. We also discuss more general models of margining investment portfolios in batches.

The area of margin calculations for investment portfolios is related to a rich combinatorial structure that is not well studied but has a very important application in the financial services industry. We believe that this paper is just an invitation to join us in this intriguing research adventure.