Trajectory optimization for space launcher problem: Hamilton-Jacobi approach

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In this work we study a climbing problem for space launcher by using the Hamilton-Jacobi-Bellman (HJB) approach. It is a joint work with O. Bokanowski (Univ. Paris 7) and E. Cristiani (Univ. Roma 1).

We aim at steering the launcher to the GTO orbit with minimal propellant consumption, and under aerodynamic pressure constraint.

The classical methods used for optimization trajectory of space launchers are generally based on total discretization of the problem or on shooting algorithms. These methods are known to be very accurate. However, their use presents several difficulties. In particular, the initialization of these methods is in general a hard task. Moreover, it is often required to have a good knowledge of the global behavior of the solution before calculating it (existence of singular arcs, number of commutations and so on).

Here, we investigate the potential of the HJB approach for trajectory optimization in the case of European Ariane 5 launcher. The corresponding control problem involves 7 state variables. We shall assume that the launcher evolves in a plane (equatorial plane). This assumption leads to a simplified model involving only 4 state variables. Despite of this simplification, there remain many theoretical and numerical difficulties that must be dealt with.

First, the control problem is in presence of state constraints. For this kind of problems, the characterization of the value function has only been

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studied under some strong controlability properties ("Inward pointing condition" introduced by Soner [6], or "outward pointing condition" studied by Frankowska [5]). Here, we give a new characterization of the value function without any controlability assumption (see also [1]).

The HJB equation is solved by the Ultra Bee scheme [4] which is known for its non-diffusive property. To speed up the computation and save memory we use the storage technique developed in [3]. It consists in storing in a special dynamic data structure only a subset of the grid nodes at each time step and compute the solution only at these nodes. This technique allows to have a fast and efficient algorithm which allow to deal with the climbing problem in a reasonable time. Numerical tests are performed on a real problem given by CNES[1] [2].

References

[1] M. I. Bardi and I. Capuzzo-Dolcetta. *Optimal Control and viscosity solutions of Hamilton-Jacobi-Bellman equations*. Birkhäuser Boston, 1997.

[2] O. Bokanowski, E. Cristiani, J. Laurent-Varin, and H. Zidani. Hamilton-Jacobi-Bellman approach for the climbing problem. *In preparation*, 2009.

[3] O. Bokanowski, E. Cristiani, and H. Zidani. An efficient data structure to solve front propagation problems. *submitted*, 2008.

[4] O. Bokanowski, N. Megdich, and H. Zidani. Convergence of a non-monotone scheme for Hamilton-Jacobi-Bellman equations with discontinuous data. *Preprint Hal, inria-00193157, submitted*, 2008.

[5] F. Frankowska. Lower semi-continuous solutions of hamilton-jacobi-equations. *SIAM J.Control Optim.*, 31:257–272, 1993.

[6] H. M. Soner. Optimal control with state space constraint. *SIAM Journal of Control and Optimization*, 24(3):552–561, 1986.

[1] French space agency