Necessary Conditions for Optimal Control Problems Governed by a Volterra Integral Equation

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Necessary conditions of optimal control problems involving integral equations have received the attention of several authors, see for example Bakke (Ref. [1]) and Carlson (Ref. [2]). More recently, models including nonlinear time delay in the state variable were formulated by Burnap and Kazemi (Ref. [3]).

In the present talk we consider the optimal control problem as follows:

$$\max_{(T,u,x)} J\left(T,u,x\right) = \int_0^T L\left(t, x\left(t\right), u\left(t\right)\right) dt + g(x(T)) \qquad u(t) \in \mathcal{U},$$

where the system is governed by a Volterra integral equation

$$x(t) = h\left(t\right) + \int_0^t f\left(t, s, x\left(s\right), u\left(s\right)\right) ds \qquad t \in [0,T],$$

and constraints depend on the terminal time and state:

$$\phi\left(T, x\left(T\right)\right) = 0,$$
$$\psi\left(T, x\left(T\right)\right) \leq 0.$$

The control $u^*\left(t\right)$ defined in the interval $[0,T^*]$ is optimal if the triple (T^*, x^*, u^*) maximizes J over all of admissible triples (T, x, u).

We establish necessary conditions for the optimal terminal time and control and new transversality conditions for the optimal time using Pontryagin-type perturbations.

Further, following the ideas of Bonnans (see [4] where it is proved a maximum principle for problems governed by an ordinary differential equation) and using Ekeland's variational principle we intend to give necessary conditions for an optimal control problem governed by a Volterra integral equation with restrictions on the state for all t, i.e.

$$\psi\left(x\left(t\right)\right) \leq 0; \quad t \in (0,T).$$

References

[1] BAKKE, V. L., *A Maximum Principle for an Optimal Control Problem with Integral Constraints,* Journal of Optimization Theory and Applications, Vol. 13, pp. 32-55, 1974.

[2] CARLSON, D. A., *An Elementary Proof of the Maximum Principle for Optimal Control Problems Governed by a Volterra Integral Equation*, Journal of Optimization Theory and Applications, Vol. 54, pp. 43-61, 1987.

[3] BURNAP, C., and KAZEMI M., *Optimal Control of a System Governed by Nonlinear Volterra Integral Equations with Delay,* IMA Journal of Mathematical Control and Information, Vol. 16, pp. 73-89, 1999.

[4] BONNANS, J.F., *Aspects de l'optimisation continue,* Notes de cours, Master OJME. Révision du 27 janvier 2008.