Fluid flows with slip boundary conditions

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ABSTRACT

At the interface between a fluid and a solid wall, it is generally assumed that the fluid satisfies the no-slip boundary condition. This assumption is being more and more questioned in the case of polymeric fluids and even for Newtonian fluids, particularly in highly confined geometries like capillaries or microfluidic devices, and their modeling then allows them to slip at this interface. On the other hand, slip boundary conditions are frequently encountered in the modeling of turbulent fluid flows through so called *wall boundary conditions*. The numerical approximation of fluid flow equations equipped with slip boundary conditions are then of fundamental importance for both modeling and optimization purposes.

While the finite element approximation of Stokes or Navier-Stokes equations with no-slip boundary conditions has been largely analyzed and experimented computationally, slip boundary conditions have paid much less attention and their finite element approximation may lead to surprising results.

The simplest slip boundary condition is the so-called *free slip* condition

$$\boldsymbol{u} \cdot \boldsymbol{\nu} = 0 \quad \text{on } \Gamma, \tag{1}$$

$$\boldsymbol{\nu} \cdot \boldsymbol{T}(\boldsymbol{u}, p) \cdot \boldsymbol{\tau} = 0, \quad \text{on } \Gamma. \tag{2}$$

Here Γ is the part of the fluid domain boundary where slip occurs, $\boldsymbol{u}$ and p stand for velocity and pressure respectively, $\boldsymbol{T}(\boldsymbol{u}, p)$ is the stress tensor, $\boldsymbol{\nu}$ and $\boldsymbol{\tau}$ are the outward unit normal vector and a tangent unit vector along Γ respectively.

Depending on the weak formulation of the fluid flow equations, the slip boundary conditions can be enforced strongly or weakly (like with a multiplier technique, or using Nitsche's method). In this talk, we shall present our numerical investigations using several types of strategies. In particular we shall show that, in the case of a curved interface, some choices of the weak formulation and the finite element approximation space may lead to divergent approximation methods, particularly when the flux condition $\boldsymbol{u} \cdot \boldsymbol{\nu} = 0$ is imposed in a weak sense.

The finite elements that are tested are defined on a mesh built over a polygonal domain approaching the (hence, non-polygonal) domain with curved boundary Γ. Denoting by h the corresponding mesh size, the polygonal domain Ω_h induces an outward unit normal $\boldsymbol{\nu}_h$ which is defined at almost every point in Γ_h. At every corner of the polygonal boundary, several choices are possible, like $\boldsymbol{\nu}$ itself, or any approximation based on the values of $\boldsymbol{\nu}_h$ at adjacent element faces. Several strategies are studied, and convergence or divergence for several finite element approximations are shown, depending on this choice, in the case where the flux condition is imposed in a strong sense.