On semi-infinite optimization problems with elliptic PDEs

Pedro Merino[†], **Ira Neitzel**[‡], **Fredi Tröltzsch**[‡]

[†]*Department of Mathematics, EPN Quito, Ecuador*

[‡]*Institute of Mathematics, TU Berlin*

In many applications of PDE constrained optimization, pointwise inequalitites must be imposed on the state of the system. On the other hand, often the control function depends only on finitely many real control parameters. We mention applications to cooling steel, to sublimation processes of crystal growth, or to the local hyperthermia in cancer therapy. Such control problems belong to the class of semi-infinite optimization, where the theory is already fairly rich.

For instance, first- and second-order optimality conditions are well known since years. However, the considerations were focussed mainly on optimization problems, where the objective functional and the constraints can be expressed in terms of explicitely given real functions. In PDE constrained optimization, these functions are defined in an implicit way via partial differential equations.

Therefore, their numerical analysis rises new questions. For instance, adjoint equations with measures on the right-hand side appear in a natural way. Moreover, and this is the main issue of the talk, the numerical solution of the problems must include a discretization of the partial differential equation by finite element or finite difference methods.

We consider the question of error estimates for the optimal solutions of semi-infinite optimization problems for elliptic equations under a finite element approximation of the PDE. The finitely many control parameters are subject to box constraints, while the state function is constrained in all points of the underlying domain.

We estimate the error of the associated discretized optimal solution and compare the result with the case of state constraints in only finitely many points that we studied in an earlier work. Numerical tests confirm our theoretical results.