Finite element error estimates for Neumann boundary control problems on graded meshes

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In this talk we will discuss a-priori error estimates for a specific elliptic linear-quadratic optimal control problem in 2D with Neumann boundary control and inequality constraints on the control variable. The domain is assumed to be polygonal and maybe non-convex. The approximations of the optimal solution are constructed in a postprocessing step by a projection of the discrete adjoint state. Although the quality of these approximations is in general affected by the appearance of corner singularities, we will show that the order of convergence can be improved provided the mesh is sufficiently graded.

Several articles treat Neumann boundary control problems, e.g. [4, 5, 6], but all of them have in common that the underlying domain is either convex or the boundary is smooth. To our knowledge there is only one published result concerning Neumann boundary control problems in polygonal concave domains. In [7] the optimal solutions are calculated in a postprocessing step, and among other things approximation rates on quasiuniform meshes are given for non-convex domains.

Optimal error estimates for optimal control problems with distributed control in non-convex domains have already been derived in [1, 2, 3] using postprocessing and graded meshes.

We will discuss how to integrate these techniques into the postprocessing approach of [7] for the case of Neumann boundary control problems in non-convex domains to improve the order of convergence.

To this end, we consider the model problem

$$\min\ J(y,u) = \frac{1}{2}\|y-y_d\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|u\|^2_{L^2(\partial\Omega)}$$

subject to

$$\begin{aligned} -\Delta y + y &= 0 \quad \text{in } \Omega,\\ \partial_\nu y &= u \quad \text{on } \partial\Omega, \end{aligned}$$

and $u \in U_{ad}$, with

$$U_{ad} = \left\{ u \in L^2(\partial\Omega)\ :\ u_a \le u \le u_b \text{ a.e. on } \partial\Omega \right\}.$$

For the approximation of this problem we use a finite element discretization with piecewise linear functions for the state and the adjoint state. The control is discretized by piecewise constant functions. Based on this discretization, we derive L^2-error estimates depending on the mesh grading parameter μ.

The quality of the approximations of the optimal control problem is demonstrated by numerical examples.

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