

# On a Strategy of Solving Parabolic Optimal Control Problems

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In this talk we present a strategy to solve parabolic optimal control problems using available specialized elliptic PDE solvers. We aim at an indirect solution approach, i.e. developing optimality conditions in function spaces that are then discretized and solved. Classes of problems where optimality conditions can be derived as coupled systems of parabolic partial differential equations are considered. This includes settings without inequality constraints as well as problems with regularized pointwise state constraints.

An intrinsic difficulty associated with time-dependent PDE control problems lies in the fact that the PDEs contained in the optimality system have reverse time-directions. This complicates the numerical treatment of such problems. Instead of the somewhat classical strategy of sequentially solving the forward state and the backward adjoint equation we consider a simultaneous space-time discretization. We refer to [3], where we considered both approaches for problems without inequality constraints, and to [4], where we extended the simultaneous space-time discretization also to regularized state-constrained problems.

In this talk, we verify that for our model problems the parabolic forward-backward system of PDEs can equivalently be expressed by a single elliptic boundary value problem in the space-time domain, as is mentioned in [1]. This fact has lately been used as a motivation for space-time-multigrid solution approaches for example in [2], which may also be an option in our context. However, instead of focussing on the rather finite-dimensional nature of

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solution algorithms we place the focus of our talk on the formulation of the approach in function spaces with special emphasis on regularized pointwise state constraints.

The theoretical base developed for the example problems then allows to apply specialized elliptic PDE solvers to the optimality system without much implementational effort. Numerical experiments for some example problems are conducted and underline the applicability of this approach.

## References

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