ESTIMATION OF DOMAINS OF ATTRACTION OF CHEMICAL AND BIOCHEMICAL REACTORS

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The Domain of Attraction (DOA) of an asymptotically stable equilibrium point of a dynamic system is the region of the state space where trajectories that converge to such equilibrium originate. DOAs are in general complicated sets, which does not admit analytical representation except in special cases. Estimations of the DOAs are usually desired for the proper analysis and operation of nonlinear dynamic systems and many techniques have been proposed so far to address their characterization (Genesio et al., 1985).

In particular, the Lyapunov stability theory (Khalil, 1996), which provides an energetic approach to dynamic stability analysis, is the basis of a family of techniques whose rationale is to approximate the DOA by a level set of a Lyapunov function of the system under study.

Consider that a general dynamic system d**x**/dt=**f**(**x**) has a Lyapunov function V(**x**) at the origin, being **x** the vector of states and **f**(.) general nonlinear functions. The problem of finding an estimation of the DOA of the equilibrium under study can be formulated as follows:

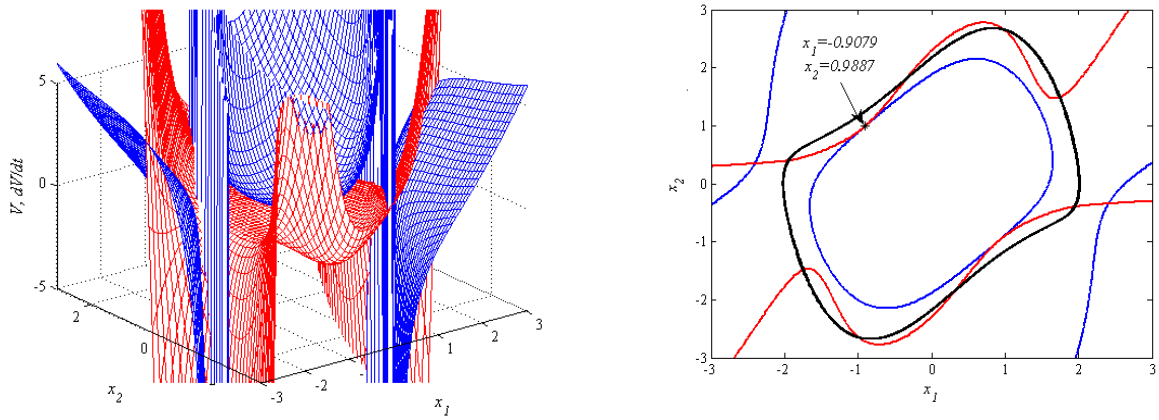
$$\begin{aligned} &\max_{c,\mathbf{x}} \quad c \\ \text{s. t.} \quad & V(\mathbf{x}) - c = 0 \qquad \text{a)} \\ & \dot{V}(\mathbf{x}) \prec 0 \qquad \text{b)} \end{aligned} \tag{1}$$

being expression (1a) the level set that estimates the DOA and expression (1b) the requirement of negative definiteness on the time derivative of the Lyapunov function.

This problem is very challenging for the general case since the underlying model is nonlinear semi-infinite. Recently (Matallana et al., 2009) proposed a global optimization approach to identify the best level set of a given Lyapunov function, which overcomes some limitations of previous approaches. The proposed formulation is based on maximal Lyapunov functions (Vanelli and Vidyasagar, 1985) and includes a set of additional constraints, which accounts for a number of necessary conditions that should be verified at the solution.

As a motivating example, in Fig. 1 it is graphically illustrated the solution of problem (1) for the classic Van der Pol oscillator (2) (Kahlil, 1996). In Fig. 1a) the adopted Lyapunov function (Vanelli and Vidyasagar, 1985) and its time derivative are shown and in Fig. 1b) the level sets of interest depicted. The closed part of the level set of V(**x**)-c=0 (blue line) constitutes an approximation of the actual DOA of the system under study (black line).

$$\frac{dx_1}{dt} = -x_2$$
$$\frac{dx_2}{dt} = x_1 - x_2(1 - x_1^2) \qquad (2)$$



a) Surfaces:
$V(\mathbf{x})$ (blue), $dV(\mathbf{x})/dt$ (red)

b) Level sets:
c = 2.2086, x_1 = -0.9079, x_2 = 0.9887

Fig. 1: Lyapunov function and time derivative for the Van der Pol system

In this contribution the proposed technique is applied to the estimation of the domains of attraction of reacting systems, which are major operation units within chemical and biochemical process industries. Specifically, a study based on the comparison of estimations with different Lyapunov functions and the actual DOA of the system will be performed. Then, the implications of the knowledge of such estimations will be analyzed in the context of the closed and open loop operations of the reactor.

References

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