SEMI-SMOOTH NEWTON METHODS FOR OPTIMAL CONTROL OF THE WAVE EQUATION

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1. INTRODUCTION

In this talk we consider the convergence properties of semi-smooth Newton methods for optimal control problems governed by the wave equation and subject to pointwise inequality control constraints. These methods can be equivalently reformulated as primal-dual-active set strategies (PDAS), see (HIK). There are several publications dealing with PDAS strategies which are very efficient for solving optimal control problems with constraints, see, e.g., (IK; BIK). For Dirichlet optimal control problems governed by a parabolic state equation superlinear convergence of PDAS was shown in (KV).

2. CONVERGENCE

We analyze the convergence properties of semi-smooth Newton methods in three different situations: optimal distributed control, optimal Neumann boundary control and optimal Dirichlet boundary control with respect to the wave equation including pointwise inequality control constraints. For distributed control and for Neumann boundary control we prove superlinear convergence of PDAS. For Dirichlet optimal control based on the very weak solution of the wave equation we show, that there is no special smoothing property allowing to prove superlinear convergence.

However, when considering the damped wave equation, i.e.

$$\begin{cases} y_{tt} - \Delta y - \rho \Delta y_t = f \quad \text{in } \Omega \times [0,T], \\ y = u \quad \text{on } \partial\Omega \times [0,T], \\ y(0) = y_0, \quad y_t(0) = y_1 \quad \text{in } \Omega, \\ \varphi \le y \le \psi \quad \text{on } \partial\Omega \times [0,T], \end{cases}$$

with state $y \in L^2(\Omega \times [0,T])$, control $u \in L^2(\partial\Omega \times [0,T])$, bounds $\varphi, \psi \in L^2(\partial\Omega \times [0,T])$ and damping parameter $\rho > 0$, $\rho \in \mathbb{R}$ we can prove superlinear convergence.

3. NUMERICAL EXAMPLES

A discretization based on space-time finite elements is proposed and numerical examples are presented which show the superlinear convergence of PDAS in the case of distributed control and Neumann boundary control as well as in the case of Dirichlet control when considering the damped wave equation.

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