CONSTRAINED STOCHASTIC CONTROLLABILITY OF NONLINEAR SYSTEMS

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Abstract.
Controllability is one of the fundamental concepts in mathematical control theory. This is a qualitative property of dynamical control systems and is of particular importance in control theory. Systematic study of controllability was started at the beginning of sixties, when the theory of controllability based on the description in the form of state space for both time-invariant and time-varying linear control systems was worked out. Roughly speaking, controllability generally means, that it is possible to steer dynamical control system from an arbitrary initial state to an arbitrary final state using the set of admissible controls.
Controllability both for linear or nonlinear stochastic dynamical systems has been recently discussed in the literature and sufficient or necessary and sufficient conditions for stochastic controllability have been formulated and proved. However, it should be pointed out that all these results were obtained only for unconstrained controls.
In this paper, we shall consider constrained local controllability problems for finite-dimensional nonlinear stochastic dynamical control systems, described by ordinary differential state equations. More precisely, we shall formulate and prove sufficient conditions for constrained stochastic local controllability in a prescribed time interval for nonlinear stochastic dynamical systems which nonlinear term is continuously differentiable near the origin. It is generally assumed that the values of admissible controls are in a given convex and closed cone with vertex at zero, or in a cone with nonempty interior. Proof of the main result is based on a so-called generalized open mapping theorem.
Throughout this paper, unless otherwise specified, we use the following standard notations.
Let (Ω,☞,⚐) be a complete probability space with probability measure P on Ω and a filtration $\{F_t\}_{t\geq 0}$. Let w(t), t≥0 be n-dimensional Wiener process defined on the probability space (Ω,☞,⚐).
Let $L_2(\Omega, F_T, R^n)$ denotes the Hilbert space of all F_T-measurable square integrable random variables with values in R^n. Moreover, let $L_2^F([0,T], R^n)$ denotes the Hilbert space of all square integrable and F_t-measurable processes with values in R^n.
Let us consider the nonlinear finite-dimensional stochastic control system described by the stochastic ordinary differential equation

$$dx(t) = f(x(t),u(t))dt + \sigma dw(t) \qquad \text{for} \quad t \in [0,T] \tag{2.1}$$

with zero initial condition x(0)=0, where $f: R^n \times R^m \to R^n$ is a nonlinear mapping such that f(0,0)=0. It is assumed that the nonlinear mapping f is continuously differentiable near the origin with respect to both arguments, σ is n×n dimensional constant matrix, w is n-dimensional Wiener process.
Let $U \subset R^m$ be a closed convex cone in the space U with vertex at zero and nonempty interior. In the sequel for simplicity of considerations we generally assume that the set of admissible controls $U_{ad} = L_2^F([0,T], U)$.

For the stochastic dynamical system (2.1) it is possible to define many different concepts of controllability. In the sequel we shall focus our attention on constrained stochastic controllability in a given time interval [0,T]. In order to do that, first of all let us introduce the notion of the attainable or reachable set at time T > 0 from zero initial state x(0) = 0, denoted by $R_T(U)$ and defined as the set of all states reachable from initial state x(0)=0 in time T>0, using admissible controls

$$R_T(U_{ad}) = \{x(T;0,u) \in L_2(\Omega, F_T, R^n) : u \in U_{ad}\}$$

where x(t,0,u), (t>0) is the unique solution of the equation (2.1) with zero initial condition and control $u \in U_{ad}$.

Definition 2.1 The dynamical system (2.1) is said to be U-locally stochastic controllable in [0,T] if the reachable set $R_T(U)$ contains a neighborhood of zero in the space $L_2(\Omega,F_T,R^n)$.

Definition 2.2 The dynamical system (2.1) is said to be U-globally stochastic controllable in [0,T] if reachable set $R_T(U_c) = L_2(\Omega,F_T,R^n)$.

It is convenient to study constrained controllability problems for the nonlinear stochastic dynamical system (2.1) with the aid of its associated linear approximation near origin of the following form

$$dz(t) = (Fz(t) + Gv(t))dt + \sigma dw(t) \qquad \text{for } t \in [0,T] \qquad (2.3)$$

with zero initial condition z(0)=0, where F and G are $n \times n$ and $n \times m$ constant matrices given by the following equality

$$F = D_1 f(x(t,0,u),u(t)) \qquad G = D_2 f(x(t,0,u),u(t))$$

Here $D_1 f$ and $D_2 f$ are the partial derivatives of the nonlinear mapping f with respect to the first and second arguments of function f respectively.

In order to compare constrained controllability results for nonlinear and associated linear dynamical system, we need the following general theorem.

Theorem 3.1 Suppose that
(i) f(0,0) = 0,
(ii) U is a closed convex cone with vertex at zero,
(iii) The linear dynamical system (2.3) is U-globally stochastic controllable in [0,T].
Then the nonlinear dynamical system (2.1) is U-locally stochastic controllable in [0,T].

References.

1. Klamka J., Controllability of Dynamical Systems, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1991.
2. Klamka J., Controllability of dynamical systems - a survey, Archives of Control Sciences, vol.2, no.3/4, 1993, pp.281-307.
3. Klamka J., Constrained controllability of nonlinear systems, Journal of Mathematical Analysis and Applications, vol.201, no.2, 1996, pp.365-374.
4. Klamka J., Schauder's fixed point theorem in nonlinear controllability problems, Control and Cybernetics, vol.29, no.3, 2000, pp.377-393.