Interior point methods in function space for state constraints - inexact Newton and adaptivity

Andreas Günther*, Anton Schiela†

Abstract. We consider an interior point method in function space for PDE constrained optimal control problems with state constraints. Our emphasis is on the construction and analysis of an algorithm that integrates a Newton path-following method with adaptive grid refinement. The algorithm consists of three nested loops: a path-following scheme, a Newton corrector, and the approximate solution of an operator equation. The crucial point is that the two outer loops are performed inexactly in function space. Discretization only takes place in the innermost loop such that the discretization error (considered as perturbation in function space) of each Newton step is controlled by adaptive grid refinement. As a consequence our algorithm allows to perform most of the required Newton steps on coarse grids, such that the overall computational time is dominated by the last few steps. In order to have a concrete class of problems at hand we consider optimal control problems subject to an elliptic partial differential equation. Then the adaptive refinement process is steered by an a-posteriori error estimator designed to estimate the error in function space in a problem suited norm. To highlight the desired property of our algorithm in terms of operating on coarsest possible grids we investigate its performance by some numerical examples.

*University of Hamburg, Germany, andreas.guenther@math.uni-hamburg.de
†Zuse Institute Berlin, Germany, schiela@zib.de