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MATHEMATICAL ASPECTS EXISTING IN COMPLEXITY OF EUROPEAN RAILWAY NETWORK

1. Introduction

This paper analyses the complexity of European Railway Network, using (ANs) networks (**A**-**N**etwork**s**). This is based on networks theory (ANs), on examples of those and also on theory of systems complexity analysed by networks (Sorin Baiculescu, 1999-2009), extracting conclusions.

2. General aspects regarding the networks of hierarchical multidimensional evolution (ANs)

Networks (ANs) are hierarchical multidimensional evolution networks, having different ranks. Their complexity varies horizontally on the same rank, or vertically from an inferior rank to a superior one. Hierarchically character, evolutive and multidimensional of networks (ANs) is it created by rank of sets and component subsets, the largest set including the set of evolutive relations. Multidimensionality is it understood as a number of criterias used to classify the components of the same rank. Sets of inferior rank do constitute the subset of rank zero which is belonging of the set immediately superior.

3. Applying network graphs (ANs) into contextual analysis of European Railway Network

The Networks (ANs) belonging of European Railway Network are: $ANs^{(1)}$ - associate of Regional Railway Network (k=1), $ANs^{(2)}$ - associate of National Railway Network, (k=2); $ANs^{(3)}$ - associate of European Railway Network (k=3). Those networks have an hierarchically character on horizontal inside the same rank and vertically from a lower rank to a higher one. The meaning of signs <i,j> is it shown in separate paragraph. Between $ANs^{(k)}$ Networks there are inclusion relationship $ANs^{(1)}\subset ANs^{(2)}\subset ANs^{(3)}$ (6) the network of lower rank forms subset of rank zero which is belonging to the immediately superior rank ($S_{k+1,0}=ANs^{(k)}$, k=1,2). The phenomens which are taking place inside the networks are highlighted by phenomenological functions (named also operators) noted $f_i^{j(k)}$, with application on subsets $S_i^{j(k)}$, values of them belonging to domains $\sigma_i^{j(k)}$, (i=0,1,2,3; j=0,1,2,…,p; k=1,2,3); $f_i^{j(k)} : S_i^{j(k)} \rightarrow \sigma_i^{j(k)}$ (1). For the whole set $S_i^{(k)}$:

$$h_i^{(k)} = \sum_{j=o}^{p} I_i^{j(k)} = \sum_{j=o}^{p} \int_{S_i^{j(k)}} f_i^{j(k)} dS_i^{j(k)} \quad (2)$$

in care $I_i^{j(k)} = \int_{S_i^{j(k)}} f_i^{j(k)} dS_i^{j(k)}$ (3) is quantifying the effect created by the operator $f_i^{j(k)}$ from subset $S_i^{j(k)}$

Variation mode of index k, p, i, is shown in Table 1.

k→	1	2	3
i ↓	p		
0	5	2	2
1	3	1	1
2	2	2	2
3	4	4	4

Tab.1
Variation of index k,p,i inside relations (1),(2),(3)

Phenomenologically functions $\varphi_i^{j,j+1(k)}$ si $\varphi_{i+1,i}^{(k)}$ describe horizontally evolution $(H_i^{j,j+1})^{(k)}$ inside of a set and vertically evolution $(V_{i,i+1})^{(k)}$ (inter-networks, inter-sets) which are taking place inside the $\Omega_i^{j,j+1(k)}$ the respective domain $\Omega_{i,i+1}^{(k)}$ having values in domains $\omega_i^{j,j+1(k)}$; $\omega_{i,i+1}^{(k)}$:

$$\varphi_i^{j,j+1(k)} : \Omega_i^{j,j+1(k)} \rightarrow \omega_i^{j,j+1(k)} \quad (4) \quad ; \quad \varphi_{i+1,i}^{(k)} : \Omega_{i,i+1}^{(k)} \rightarrow \omega_{i,i+1}^{(k)} \quad (5)$$

Lots $\Omega_i^{j,j+1(k)}$ and $\Omega_{i-1,i}^{(k)}$ establish interfaces between lots $\sigma_i^{j(k)}$, $\sigma_i^{j+1(k)}$ respectively $\omega_{i-1,i}^{(k)}$, $\omega_{i,i+1}^{j,j+1(k)}$. Inside them are taking place phenomenoms of transfer, diffusion, fluctuation, and also exchanges (continues/variable) of situations, generated by energy and information. The effect of the phenomenologically functions $\varphi_i^{j,j+1(k)}$, $\varphi_{i+1,i}^{(k)}$ is quantified through:

$$I_i^{j,j+1(k)} = \left(\int_{\Omega_i^{j,j+1(k)}} ((\partial\varphi_i^{j,j+1(k)}/\partial x)dx + (\partial\varphi_i^{j,j+1(k)}/\partial\tau)d\tau)\right) + Ci^{j,j+1(k)} \quad (6)$$

$$I_{i,i+1}^{(k)} = \left(\int_{\Omega_{i,i+1}^{(k)}} ((\partial\varphi_{i,i+1}^{(k)}/\partial x)dx + ((\partial\varphi_{i,i+1}^{(k)}/\partial\tau)d\tau)\right) + C_{i,i+1}^{(k)} \quad (7)$$

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x - vector associated to spatial coordinates; τ - time. General form of operators which are taking place inside subsets $S_i^{j(k)}$ belonging to sets $S_i^{(k)}$ of networks $ANs^{(k)}$ and inside interfaces $I_i^{j,j+1(k)}$ si $I_{i-1,i}^{(k)}$ in shown in (Fig.1):

	j		j+1	
	$f_i^{j(k)}$ $S_i^{j(k)} \rightarrow \sigma_i^{j(k)}$	$\varphi_i^{j,j+1(k)}$ $\Omega_i^{j,j+1(k)} \rightarrow \omega_i^{j,j+1(k)}$ $I_i^{j,j+1(k)}$ <INTERFACE>	$f_i^{j+1(k)}$ $S_i^{j+1(k)} \rightarrow \sigma_i^{j+1(k)}$	i
		<INTERFACE> $\varphi_{i-1,i}^{(k)}$ <INTERFACE> $\Omega_{i-1,i}^{(k)} \rightarrow \omega_{i-1,i}^{(k)}$ $I_{i-1,i}^{(k)}$		
	$f_{i-1}^{j(k)}$ $S_{i-1}^{j(k)} \rightarrow \sigma_{i-1}^{j(k)}$	$\varphi_{i-1}^{j,j+1(k)}$ $\Omega_{i-1}^{j,j+1(k)} \rightarrow \omega_{i-1}^{j,j+1(k)}$ $I_{i-1}^{j,j+1(k)}$ <INTERFACE>	$f_{i-1}^{j+1(k)}$ $S_{i-1}^{j+1(k)} \rightarrow \sigma_{i-1}^{j+1(k)}$	i-1

Fig.1 Functional operators inside subsets $S_i^{j(k)}$ belonging sets $S_i^{(k)}$ of network $ANs^{(k)}$ and inside interfaces $I_i^{j,j+1(k)}$ and $I_{i-1,i}^{(k)}$

Complexity of network $ANs^{(k)}$ has expression : $C_{ANs}^{(k)} = A^{j(k)} + B^{(k)} + A^{p(k)}$ (8), in which:

$$A^{j(k)} = \sum_{i=0}^{3} \sum_{j=0}^{p-1} \Big[\int_{S_i^{j(k)}} f_i^{j(k)} dS_i^{j(k)} + \Big(\int_{\Omega_i^{j,j+1(k)}} ((\partial\varphi_i^{j,j+1(k)}/\partial x)dx + (\partial\varphi_i^{j,j+1(k)}/\partial\tau)d\tau)\Big) + Ci^{j,j+1(k)} \Big] \quad (9)$$

$$B^{(k)} = \sum_{i=0}^{2} \Big[\Big(\int_{\Omega_{i,i+1}^{(k)}} ((\partial\varphi_{i,i+1}^{(k)}/\partial x)dx + ((\partial\varphi_{i,i+1}^{(k)}/\partial\tau)d\tau)\Big) + C_{i,i+1}^{(k)} \Big] \quad (10)$$

$$A^{p(k)} = \sum_{i=0}^{3} \int_{S_i^{p(k)}} f_i^{p(k)} dS_i^{p(k)} \quad (11)$$

;

Index k, p, i from expressions (8) ÷ (11) are shown in Table 1.

Cooperative and hierarchical system of networks ANs is characterised by formal differentially nonlinear equations, of horizontally evolution : $s_i^{j,j+1(k)}(x,\tau) = \varphi_i^{j,j+1(k)}(x,\int\ldots,\nabla,\alpha,\tau)$ (12), and vertically evolution: $s_{i,i+1}^{(k)}(x,\tau) = \varphi_{i,i+1}^{(k)}(x,\int\ldots,\nabla,\alpha,\tau)$ (13) having solutions:

$s_i^{j,j+1(k)}(x,\tau) = S_{0,i}^{j,j+1(k)} + \sum_{st.} \varepsilon_{st.,i}^{j,j+1(k)}(\tau)\omega_{st.}^{j,j+1(k)}(x) + \sum_{inst.} \varepsilon_{inst.,i}^{j,j+1(k)}(\tau)\omega_{inst.}^{j,j+1(k)}(x)$ (14), respectively:

$s_{i,i+1}^{(k)}(x,\tau) = S_{0i,i+1}^{(k)} + \sum \varepsilon_{st.,i,i+1}^{(k)}(\tau)\omega_{st.i,i+1}^{(k)}(x) + \sum \varepsilon_{inst.,i,i+1}^{(k)}(\tau)\omega_{inst.i,i+1}^{(k)}(x)$ (15), making notes on signification of indexes used. The parameters used in relations (13), (14), (15) were the following: $s_i^{j,j+1(k)}$ - the function of status parameters of horizontally evolution processus; $s_{i,i+1}^{(k)}$ - the function of status parameters of vertically evolution processus, x – vector associated to spatial coordinates; τ - time; φ - nonlinear function; $\int\ldots$ integration (is it showing the global character of cooperative and enslavement of subsystems); ∇ - operator (is it flows (diffusions)); α – vector associate to order parameters; st. - stable regime; inst. – instable regime; S_0 - particular value of vector α

Conclusions:

This paper established the form of hierarchical evolutive multidimensional networks attached to European Railway Network, establishing the complexity and nonlinear aspects of it's functional dynamics (nonlinear differential equations). Between networks $ANs^{(3)}$, $ANs^{(2)}$, $ANs^{(1)}$ attached to European Railway Network there are systemic dependence and also relations of cybernetics settlement of type $ANs^{(1)} \Leftrightarrow ANs^{(2)} \Leftrightarrow ANs^{(3)}$ (feedback / feedbefore). The correct functioning of all ERN network, with his attached form AN_s, involves functional simultaneity for all his components. On estimation of $ANs^{(k)}$ Network complexity , phenonmenologically functions involves quantitative and qualitative components (marked by indexes C, Q), $f_i^{j(k)} = (f_i^{j(k)C}, f_i^{j(k)Q})$; $\varphi_i^{j,j+1(k)} = (\varphi_i^{j,j+1(k)C}, \varphi_i^{j,j+1(k)Q})$; $\varphi_{i,i+1}^{(k)} = (\varphi_{i,i+1}^{(k)C}, \varphi_{i,i+1}^{(k)Q})$,establishing synergetic and informational character of processes. The sums $\sum$. will be marked $(^{(C)}\sum,^{(Q)}\sum)$ simultaneity of phenomens being marked by <$\wedge$ >. In the following annexe is it showing the actual **E**uropean **R**ailway **N**etwork, marked by ERN, establishing and spotlighting the main components belonging to AN_s calculation form, presented in this paper.

ANNEXE

RAILWAY NET-WORK (level)	k ↑	p ↑	j ↑	FUNDAMENTAL COMPONENTS (i=0)	INTERNAL RELATIONS (i=1)	EXTERNAL RELATIONS (i=2)	EVOLUTION RELATIONS (i=3)	$ANs^{(k)}$ ↑ (i=4)	EUROPEAN RAILWAY NETWORK (37 European countries) (i=5)
EUROPEAN (E) (k=3)	3	4	4	dynamic complexity, complex adaptative systems	artificially intelligence, learning expert systems, structural railway science (railway level)	large railway systems inside structural science	framework relationship ($ANs^{(3)}$ interface/ pan-european corridor)	$ANs^{(3)}$	E — EUROPEAN RAILWAY STRUCTURE
			3	feedback, feedbefore	european inter-states agreements referring to traffic / commercial segment	security in transportation	establishing/ developing of trans-european networks		
			2	structural adjustment	regulations regarding quantitative/qualitative development of ruling material used on european railway network	european cohesion (traffic/ commercial)	unitary policy inter-states for european railway transport		
			1	simultaneity	regulations regarding quantitative / qualitative development of european railway infrastructure	european jurisdiction	transport without frontiers all over Europe		
			0	$ANs^{(2)}$	fundamental components $ANs^{(3)}$	internal relations $ANs^{(3)}$	external relations $ANs^{(3)}$		
NATIONAL (N) (k=2)	2	4	4	system, input, output	european technical parameters	market demands	basic relationship (interface $ANs^{(2)}$ / $ANs^{(3)}$)	$ANs^{(2)}$	
			3	synergy	standards for railway traffic (speed, etc)	traffic /commercial adaptation	evolution and traffic		
			2	structure	european regulations for loading goods onto wagons	statistics / maintenance / ruling materiel adaptation on market demands	supervising committee for applying the railway regulations		
			1	foreseeing	standard gauge (wide, normal, small)	classification / statistics regarding railway infrastructure	existing workshops on transport sections		
			0	$ANs^{(1)}$	fundamental components $ANs^{(2)}$	internal relations $ANs^{(2)}$	external relations $ANs^{(2)}$		
REGIONAL (R) (k=1)	1	5	5	-	-	complimentary/ competitor transport	quality standards (ISO), inter-modal transport	$ANs^{(1)}$	N - R
			4	-	-	informational resources	environment protection, research development		
			3	-	expedition, traffic, commercial (freight transport,passenger transport)	financial strategy	open market, modernization, new technologies		
			2	transport operators	maintenance and repairing of ruling material	politically strategy	privatization		
			1	means of transport	maintenance, repairing of railway infrastructure, commercial exploitation of patrimony	economical/ social strategy	decentralization, reorganization		
			0	railway infrastructure	fundamental components $ANs^{(1)}$	internal relations $ANs^{(1)}$	external relations $ANs^{(1)}$		
				SUBSETS $S_i^{j(k)}$					
				SETS $S_i^{(k)}$ COMPONENTS OF RAILWAY NETWORKS OF HIERARCHICAL MULTIDIMENSIONAL EVOLUTION $ANs^{(k)}$					
i→				0	1	2	3	4	5

Hierarchical networks $ANs^{(k)}$ (k=1,2,3) (**A**–**N**etwork**s**) associated to ERN (**E**uropean **R**ailway **N**etwork)

(— input (I)–output (0) inter-networks (3/2, 2/1)), (— input (I)–output(0) intra-networks (3/3, 2/2, 1/1));1,2,3-hierarchical level of the components network