Large deviations for stochastic PDE with Lévy noise

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We will discuss how the theory of first and second order integro-PDE of Hamilton-Jacobi-Bellman type in Hilbert spaces can be used to establish large deviation principle for a class of abstract stochastic evolution equations with small Lévy noise intensities of the form

$$\begin{cases} dX_n(s) = (-AX_n(s) + F(X_n(s)))ds + G(X_n(s))dL_n(s), \\ X_n(0) = x \in H, \end{cases}$$

where A is a linear, densely defined maximal monotone operator in a real, separable Hilbert space H and $L_n(t)$ is a sequence of Lévy processes of the form $L_n(t) = \frac{1}{n}L(nt)$, where L is a square integrable Lévy martingale in H. The above abstract stochastic differential equation is very general and includes for instance various semilinear stochastic PDE with small Lévy noise. The key ingredient in our procedure is the use of viscosity solutions to obtain the so called Laplace limit for the large deviation problem at a single time. Optimal control plays an important part in our analysis. PDE approaches to large deviations of stochastic partial differential equations have been recently developed by Feng, Feng and Kurtz, and Święch. The results presented here are the first attempt to extend such methods to processes driven by Lévy noise and develop a theory of viscosity solutions of integro-PDE in Hilbert spaces. This is a joint work with Jerzy Zabczyk.