Approximation of Boundary Control Problems on Curved Domains. The Neumann Case

Eduardo Casas

Dpto. de Matemática Aplicada y Ciencias de la Computación, E.T.S.I. Industriales y de Telecomunicación, Universidad de Cantabria, 39005 Santander, Spain, e-mail: eduardo.casas@unican.es

ABSTRACT

In this talk we consider the following optimal control problem

$$(\mathrm{P}) \left\{ \begin{array}{l} \min J(u) = \displaystyle\int_\Omega L(x, y_u(x))\, dx + \frac{N}{2} \int_\Gamma u^2(x)\, d\sigma(x) \\ \text{subject to } \ (y_u, u) \in (L^\infty(\Omega) \cap H^1(\Omega)) \times L^2(\Gamma), \\ \alpha \leq u(x) \leq \beta \quad \text{for a.e. } \ x \in \Gamma, \end{array} \right.$$

where Γ is a smooth manifold, y_u is the state associated to the control u, given by a solution of the Neumann problem

$$\left\{ \begin{array}{rcll} -\Delta y + a(x, y) & = & 0 & \text{in } \Omega, \\ \partial_\nu y & = & u & \text{on } \Gamma. \end{array} \right. \tag{1}$$

To solve numerically this problem it is necessary to approximate Ω by a new domain (typically polygonal) Ω_h. Our goal is to analyze the effect of this change on the optimal control. More precisely, if we define a new optimal control problem in Ω_h, denoted by (P_h), we study the convergence of global or local solutions of problems (P_h) to the corresponding local or global solutions of (P) when the parameter h tends to zero. We also derive some error estimates. We do not consider the numerical discretization of (P), we just consider a family of infinity dimensional control problems (P_h) defined in Ω_h and we compare the solutions of these problems with the solutions of (P). In this way, we are studying the influence of a small change in the domain on the solutions of the control problem.