

# Singular Arcs on the Continuous Time Optimal Hydrothermal Scheduling

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We consider an optimal control problem of optimal hydrothermal scheduling. On a weekly basis, optimal hydrothermal scheduling is usually performed using linear programming techniques, see [2] and the references therein. In such a case limitations due to switch on-off of engines as well as the variable efficiency as function of height of water are neglected. Our proposal is to study the hydrothermal scheduling by taking into account this variable efficiency with respect to the volume of water in dams.

Our model is deterministic and with continuous time, which is meaningful for a short-term horizon. In this respect we follow the model in [1]. These authors obtain existence and also uniqueness of minimizers in some special situations. The thermal cost is a strongly convex and nondecreasing function of the thermal power.

In this work, we apply the tools of optimal control theory. We focus on the analysis of singular arcs. Our main result is a characterization of the Goh-Legendre (GL) condition [3, 4]. As a consequence we show that for some choices of the efficiency coefficients, this condition always holds (resp. does not hold). When the GL condition holds, the algebraic variables (controls) can be eliminated from some algebraic expressions and expressed as functions of the differential variables (state and costate).

Thus, we are able to give conditions under which these conditions either automatically satisfied or excluded.

We present numerical examples that support those results.

## References

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