Augmented Lagrangians for global optimization

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December 2, 2008

Global optimization has ubiquitous applications in all branches of engineering sciences and applied sciences. The Augmented Lagrangian methodology based on the Powell-Hestenes-Rockafellar [6, 9, 10] formula has been successfully used for defining practical nonlinear programming algorithms [3, 5]. Convergence to KKT points was proved using the Constant Positive Linear Dependence constraint qualification [2], which strengthens the results based on the classical regularity condition [4, 5].

In this work, we consider the Augmented Lagrangian method introduced in [3] and we modify it in such a way that, at each outer iteration k, we find an ε_k*-global* minimizer of the subproblem. In the definition of the subproblem we introduce an important modification with respect to [3]: besides the lower level constraints we include constraints that incorporate information about the global solution of the nonlinear programming problem. A theorem of convergence to ε-global minimizers is presented.

In the implementation, we consider linear constraints on the lower-level set, and additional valid linear constraints which result from outer approximations of the feasible region and hence incorporate the global optimum information. This allows us to use the αBB [1] method and its convex underestimation techniques [7, 8] for the subproblems, in such a way that the underestimation techniques are applied just to the Augmented Lagrangian function and not to the constraints. The αBB global optimization approach has been applied to various problems that include molecular conformations in protein folding, parameter estimation and phase equilibrium. Mixed-integer nonlinear models arising in process synthesis, design and operations problems represent additional important application areas.

There exist many global optimization techniques for nonlinear programming problems. However, up to our knowledge, the proposed approach is the first practical deterministic global optimization method based on the Augmented Lagrangian framework. Moreover, as a consequence of using the Augmented Lagrangian approach combined with the αBB method and its convex

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α-underestimation techniques, the method introduced in this paper does not rely on the specific form of the functions involved in the problem definition (objective function and constraints), apart from their continuity and differentiability. Interval arithmetic is used to compute bounds on the objective function and to compute the convex α-underestimators. Although the method can take advantage of known underestimators and relaxations for several kinds of functional forms, it can also deal with functional forms for which underestimators and relaxations have not been developed yet. In this sense, the method does not depend on the analysis of expressions involved in the problem definition to identify functional forms for which ad-hoc underestimators are available.

Convergence to ε-global minimizers is proved and numerical results are given.

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